

# Model independent sum rules for strange form factors

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We study chiral models with SU(3) group structure such as Skyrmion and chiral bag to yield theoretical predictions of proton strange form factor comparable to the recent experimental data of the SAMPLE Collaboration. For these predictions we formulate model independent sum rules for proton strange form factor in terms of baryon octet magnetic moments. We also investigate the Becci-Rouet-Stora-Tyutin symmetries associated with the Stückelberg coordinates, ghosts and anti-ghosts involved in the Skyrmion model.

## I. INTRODUCTION

There have been many interesting developments concerning the strange flavor structures in the nucleon and the hyperons. Especially, the internal structure of the nucleon is still a subject of great interest to both experimentalists and theorists. In 1933, Frisch and Stern [1] performed the first measurement of the magnetic moment of the proton and obtained the earliest experimental evidence for the internal structure of the nucleon. However, it wasn't until 40 years later that the quark structure of the nucleon was directly observed in deep inelastic electron scattering experiments and we still lack a quantitative theoretical understanding of these properties including the magnetic moments.

Recently, the SAMPLE Collaboration reported the experimental data of the proton strange magnetic form factor through parity violating electron scattering at a small momentum transfer  $Q_S^2 = 0.1 \text{ (GeV/c)}^2$  [2]

$$G_M^s(Q_S^2) = +0.14 \pm 0.29 \text{ (stat)} \pm 0.31 \text{ (sys) n.m..} \quad (1.1)$$

On the other hand, baryons were described by topological solitons [3, 4, 5, 6, 7] and the MIT bag model [8] was later unified with the Skyrmion model to yield the chiral bag model (CBM) [9], which then includes the pion cloud degrees of freedom and the chiral invariance consistently. Moreover, the soliton was exploited to yield superhaliton [10] in color flavor locking phase [11].

The QCD is the basic underlying theory of strong interaction, from which low energy hadron physics should be attainable. Moreover, for hadron structure calculations, the coupling constant  $g$  is not a relevant expansion parameter of QCD. Long ago, 't Hooft noted that  $1/N_c$  could be regarded as expansion parameter of QCD [12] where  $N_c$  is the number of colors and  $gN_c^2$  is kept constant. The properties of large  $N_c$  limit of the QCD can be satisfied by the meson sector of the nonlinear sigma model such as the Skyrmion model.

In this paper, we will study the chiral models such as the Skyrmion and chiral bag to yield theoretical predictions of proton strange form factor comparable to the recent experimental data of the SAMPLE Collaboration. To do this, we will formulate the model independent sum rules for the proton strange form factor in terms of the baryon octet magnetic moments. We will also investigate the Becci-Rouet-Stora-Tyutin (BRST) symmetries associated with the Stückelberg coordinates, ghosts and anti-ghosts involved in the Skyrmion model.

## II. BRST SYMMETRIES OF SKYRMION IN IMPROVED DIRAC QUANTIZATION

Now, in order to study the hadron physics phenomenology, we treat  $1/N_c$  as expansion parameter of QCD, so that the properties of large  $N_c$  limit of the QCD can be satisfied by the SU(3) Skyrmion model whose Lagrangian is of the form [4]

$$L = \int d^3x \left[ -\frac{f_\pi^2}{4} \text{tr}(l_\mu l^\mu) + \frac{1}{32e^2} \text{tr}[l_\mu, l_\nu]^2 \right] + L_{WZW} \quad (2.1)$$

where  $l_\mu = U^\dagger \partial_\mu U$  and  $U \in \text{SU}(3)$  is described by pseudoscalar meson fields  $\pi_a$  ( $a = 1, 2, \dots, 8$ ) and the topological aspects can be included via the WZW action [4]. Assuming maximal symmetry, we introduce the hedgehog ansatz

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$U_0$  embedded in the SU(2) isospin subgroup of SU(3) to yield the topological charge

$$Q = -\frac{1}{2\pi} \chi_E (\theta - \sin \theta \cos \theta) = 1 \quad (2.2)$$

where  $\theta$  is the chiral angle and  $\chi_E$  is the Euler characteristic being an integer two in the spherical bag surface.

In order to define the spin and isospin we can quantize, in the SU(2) Skyrmion for instance, the zero modes via

$$U_0 \rightarrow AU_0A^\dagger \quad (2.3)$$

and

$$A(t) = a^0 + i\vec{a} \cdot \vec{\tau}, \quad (2.4)$$

with  $a^\mu$  being the collective coordinates. We can then obtain the Lagrangian

$$L = -m_0 + 2i_1 \dot{a}^\mu \dot{a}^\mu \quad (2.5)$$

where the static mass  $m_0$  and the moment of inertia  $i_1$  are calculable in the Skyrmion model. Introducing the canonical momenta  $\pi^\mu$  we can obtain the canonical Hamiltonian

$$H = m_0 + \frac{1}{8i_1} \pi^\mu \pi^\mu. \quad (2.6)$$

Note that the second-class geometrical constraints

$$\begin{aligned} \Omega_1 &= a^\mu a^\mu - 1 \approx 0, \\ \Omega_2 &= a^\mu \pi^\mu \approx 0 \end{aligned} \quad (2.7)$$

should be treated via the Dirac brackets [13]. However, in the Dirac quantization, we have difficulties in finding the canonically conjugate pair, which were later overcome [14] by introducing pair of auxiliary Stückelberg fields  $\theta$  and  $\pi_\theta$  with

$$\{\theta, \pi_\theta\} = 1. \quad (2.8)$$

In the Skyrmion the first-class constraints

$$\begin{aligned} \tilde{\Omega}_1 &= a^\mu a^\mu - 1 + 2\theta, \\ \tilde{\Omega}_2 &= a^\mu \pi^\mu - a^\mu a^\mu \pi_\theta \end{aligned} \quad (2.9)$$

were constructed [15] to satisfy the strongly involutive Lie algebra

$$\{\tilde{\Omega}_1, \tilde{\Omega}_2\} = 0. \quad (2.10)$$

Similarly, the first-class Hamiltonian was formulated to yield the baryon mass spectrum

$$m_B = m_0 + \frac{1}{2i_1} \left[ J(J+1) + \frac{1}{4} \right] \quad (2.11)$$

with the isospin quantum number  $J$ . Here note that an additional global shift is due to the Weyl ordering correction. Following the BRST quantization scheme [16] with (anti)ghost and their Lagrangian multiplier fields, we obtain the BRST symmetric Lagrangian [15],

$$\begin{aligned} L_{eff} &= -m_0 + \frac{2i_1 \dot{a}^\mu \dot{a}^\mu}{1-2\theta} - \frac{2i_1 \dot{\theta}^2}{(1-2\theta)^2} - \frac{\dot{\theta} \dot{b}}{1-2\theta} \\ &\quad - 2i_1 (1-2\theta)^2 (b + 2\bar{c}c)^2 + \dot{\bar{c}}\dot{c} \end{aligned} \quad (2.12)$$

invariant under the transformations,

$$\begin{aligned} \delta_\lambda a^\mu &= \lambda a^\mu c, & \delta_\lambda \theta &= -\lambda(1-2\theta)c, \\ \delta_\lambda \bar{c} &= -\lambda b, & \delta_\lambda c &= \delta_\lambda b = 0. \end{aligned} \quad (2.13)$$

(For more details of the BRST quantization of the SU(2) and SU(3) Skyrmions, see Ref. [15] and Ref. [17], respectively.)

TABLE I: The baryon octet strange form factors

Input	$F_{2N}^s(0)$	$F_{2\Lambda}^s(0)$	$F_{2\Sigma}^s(0)$	$F_{2\Sigma}^s(0)$
CBM	0.30	0.49	0.25	-1.54
Exp	0.32	1.42	1.10	-1.10

### III. MODEL INDEPENDENT SUM RULES AND PROTON STRANGE FORM FACTORS

Next, we consider the CBM which is a hybrid of two different models: the MIT bag model at infinite bag radius on one hand and Skyrmiion model at vanishing radius on the other hand. (The explicit CBM Lagrangian is given in Ref. [7] for instance.) In the CBM the total topological charge  $Q$  in (2.2) is now splitted into the meson and quark pieces to satisfy the Cheshire cat principle [18]. Moreover, the quark fractional charge is given by sum of integer one (from valence quarks) and the quark vacuum contribution, which is also rewritten in terms of the eta invariant [19].

In the collective quantization of the CBM, we explicitly obtain the proton magnetic moment [20, 21]

$$\mu_p = \frac{1}{90}(9I_1 + 24I_2 + 12I_3 + 16I_4 - 4I_5) + \frac{2I_6}{1125}(9I_1 + 4I_2 - 8I_3) \quad (3.1)$$

with the inertia parameters  $I_n$  ( $n = 1, \dots, 6$ ) calculable in the CBM. Similarly we construct the baryon octet magnetic moments to reproduce the Coleman-Glashow sum rules [21, 22] such as  $U$ -spin symmetries,

$$\mu_{\Sigma^+} = \mu_p, \quad \mu_{\Xi^0} = \mu_n, \quad \mu_{\Xi^-} = \mu_{\Sigma^-}. \quad (3.2)$$

Now we define the Dirac and Pauli EM form factors via

$$\langle p + q | \hat{V}^\mu | p \rangle = \bar{u}(p + q) \left[ F_{1B}(q^2) \gamma^\mu + \frac{i}{2m_B} F_{2B}(q^2) \sigma^{\mu\nu} q_\nu \right] u(p) \quad (3.3)$$

where  $q$  is momentum transfer and  $\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)$  and  $m_B$  is baryon mass. The Sachs form factors are then given by

$$G_M = F_{1B} + F_{2B}, \quad G_E = F_{1B} + \frac{q^2}{4m_B^2} F_{2B}, \quad (3.4)$$

so that, at zero momentum transfer, the Pauli strange form factor is identical to the Sachs strange form factor:

$$F_{2B}^s(0) = G_M^s(0). \quad (3.5)$$

In the SAMPLE experiment, they measured the neutral weak form factor

$$G_M^{Z,p} = \left( \frac{1}{4} - \sin^2 \theta_W \right) G_M^p - \frac{1}{4} G_M^n - \frac{1}{4} G_M^s \quad (3.6)$$

with  $G_M^p$  and  $G_M^n$  being the proton and neutron Sachs form factors, to predict the proton strange form factor (1.1) which is positive value contrary to the negative values from most of the model calculations except the predictions [20, 23] of the SU(3) CBM and the recent predictions of the chiral quark soliton model [24] and the chiral perturbation theory [25, 26]. (See Ref. [7] for more details.)

In the CBM the proton strange form factor is given by [20]

$$\begin{aligned} F_{2N}^s(0) &= \frac{1}{60}(21I_1 - 4I_2 - 2I_3 - 4I_4 - 2I_5) \\ &\quad + \frac{I_6}{2250}(-129I_1 + 76I_2 - 52I_3) \end{aligned} \quad (3.7)$$

which, after some algebra with the other baryon octet strange form factors, yields the sum rule for the proton strange form factor in terms of the baryon octet magnetic moments only (for the other baryon sum rules see Ref. [27])

$$F_{2N}^s(0) = \mu_p - \mu_{\Xi^-} - (\mu_p + \mu_n) - \frac{1}{3}(\mu_{\Sigma^+} - \mu_{\Xi^0}) + \frac{4}{3}(\mu_n - \mu_{\Sigma^-}). \quad (3.8)$$

Explicitly calculating the inertia parameters  $I_n$  numerically in (3.7), we predict the proton strange form factor, 0.30 n.m. as shown in Table 1. Moreover, exploiting the experimental data for the baryon octet magnetic moments in (3.8) we obtain

$$F_{2N}^s(0) = G_M^s(0) = 0.32 \text{ n.m..} \quad (3.9)$$

On the other hand, the quantities  $G_{E,M}^Z$  in (3.6) for the proton can be determined via elastic parity-violating electron scattering to yield the experimental data  $G_M^s(Q_S^2) = +0.14 \pm 0.29 \text{ (stat)} \pm 0.31 \text{ (sys)} \text{ n.m.}$  [2] for the proton strange magnetic form factor. Here one notes that the prediction for the proton strange form factor (3.9) obtained from the sum rule (3.8) is comparable to the SAMPLE data and is shown in Table 1, together with those of the other baryon strange form factors. Moreover, from the relation (3.6) at zero momentum transfer, the neutral weak magnetic moment of the nucleon can be written in terms of the nucleon magnetic moments and the proton strange form factor [28]

$$4\mu_p^Z = \mu_p - \mu_n - 4\sin^2\theta_W\mu_p - F_{2N}^s(0). \quad (3.10)$$

#### IV. CONCLUSIONS

In conclusion, we discussed the SAMPLE experiments in the topological solitons such as the Skymion and chiral models to predict baryon strange form factors by constructing the model independent sum rules for the proton strange form factor in terms of the baryon octet magnetic moments. We also exploited the improved Dirac quantization scheme to investigate the BRST symmetries associated with the Stückelberg coordinates, ghosts and anti-ghosts involved in the Skyrme model.

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